Example 1b: Effective Properties of a Laminate

This example problem, like Example 1a, involves only determination of effective properties rather than determination of the response to an applied loading history. This problem, however, involves a cross-ply $[90°/0°]_s$ graphite/epoxy laminate rather than a unidirectional graphite/epoxy composite. Since the constituent materials are the same, the difference between the input files in this and the previous example problem amounts to replacing the **`*RUC`** keyword with the **`*LAMINATE`** keyword. While the **`*RUC`** keyword specified analysis of a continuum in Example 1a, in the present example the **`*LAMINATE`** keyword specifies analysis of the laminated plate structure (as modeled using lamination theory).

The geometry and coordinate system employed in the code for the laminate is shown in Figure 1.2. The global x-y-z coordinate system is applicable to the laminate as a whole, while the local coordinates (x_1-x_2-x_3) apply within each layer. As shown, locally, each layer is modeled using a GMC RUC analysis.

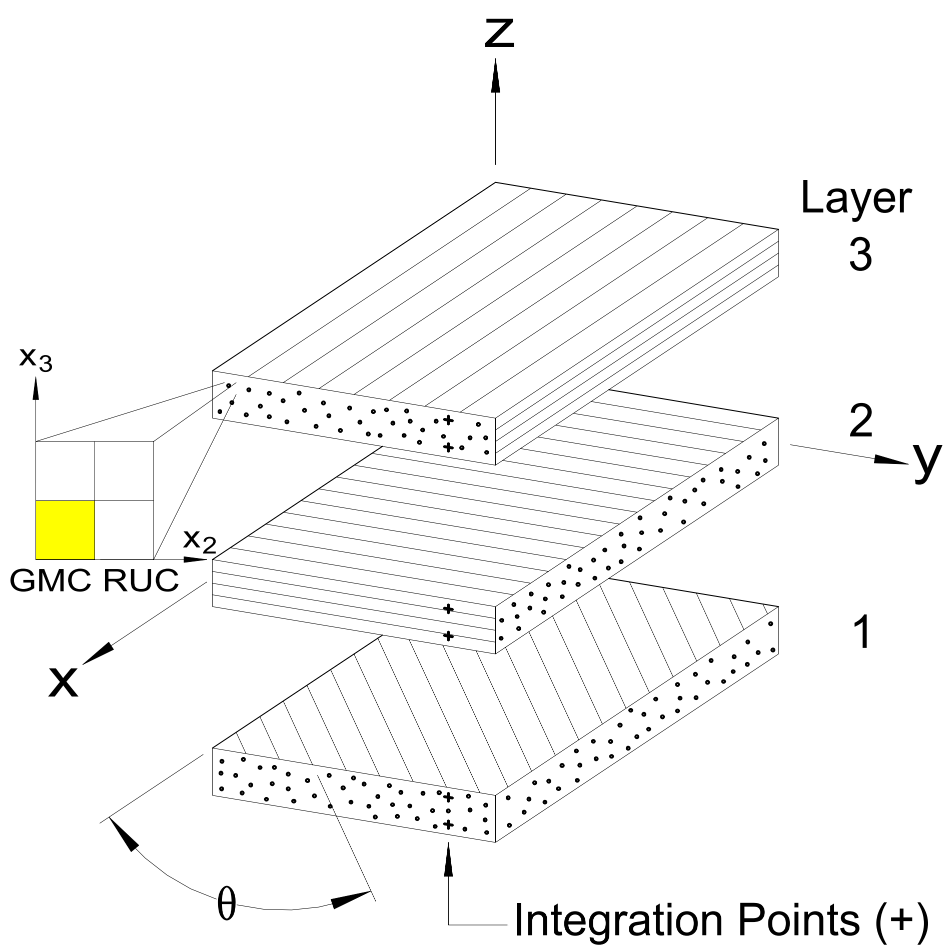

Figure 1.2 General laminate geometry and coordinate system employed in MAC/GMC 4.0. The local behavior of each layer is modeled using a generalized method of cells (GMC) repeating unit cell (RUC) analysis.

MAC/GMC Input File: `example_1b.mac`

```
MAC/GMC 4.0 Example 1b - graphite/epoxy laminate
*CONSTITUENTS
  NMATS=2
# -- Graphite fiber
  M=1 CMOD=6 MATID=U MATDB=1 &
   EL=388.2E9,7.6E9,0.41,0.45,14.9E9,-0.68E-6,9.74E-6
# -- Epoxy matrix
  M=2 CMOD=6 MATID=U MATDB=1 &
   EL=3.45E9,3.45E9,0.35,0.35,1.278E9,45.E-6,45E-6
*LAMINATE
  NLY=3
  LY=1 MOD=2 THK=0.25 ANG=90 ARCHID=1 VF=0.65 F=1 M=2
  LY=2 MOD=2 THK=0.50 ANG=0  ARCHID=1 VF=0.65 F=1 M=2
  LY=3 MOD=2 THK=0.25 ANG=90 ARCHID=1 VF=0.65 F=1 M=2
*PRINT
  NPL=-1
*END
```

Annotated Input Data

1) Flags: None

2) Constituent materials (**`*CONSTITUENTS`**) [KM_2]:

Number of materials:	2	(`NMATS=2`)
Constitutive models:	Elastic	(`CMOD=6`)
Materials:	User-defined (Graphite)	(`MATID=U`)
	User-defined (Epoxy)	(`MATID=U`)
Material property source:	Read from input file	(`MATDB=1`)
Material properties:	See Table 1.1	(`EL=`... and `K=`...)

This example problem employs the same user-defined material properties as in Example 1a.

3) Analysis type (**`*LAMINATE`**) → Laminate Analysis [KM_3]:

```
NLY=3
LY=1 MOD=2 THK=0.25 ANG=90 ARCHID=1 VF=0.65 F=1 M=2
LY=2 MOD=2 THK=0.50 ANG=0  ARCHID=1 VF=0.65 F=1 M=2
LY=3 MOD=2 THK=0.25 ANG=90 ARCHID=1 VF=0.65 F=1 M=2
```

Number of layers: 3 (`NLY=3`)

Layer	Analysis Model	Thickness	Fiber Angle	Architecture	Volume fraction	Fiber material	Matrix material
(`LY=`)	(`MOD`)	(`THK`)	(`ANG`)	(`ARCHID`)	(`VF`)	(`F`)	(`M`)
1	GMC-2D	0.25	90°	square fiber, square pack	0.65	graphite	epoxy
2	GMC-2D	0.50	0°	square fiber, square pack	0.65	graphite	epoxy
3	GMC-2D	0.25	90°	square fiber, square pack	0.65	graphite	epoxy

Because the local behavior of each layer is represented by a GMC repeating unit cell, information similar to that specified under ***RUC** (in Example 1a) is now specified for each layer. Here, each layer is represented as the same unidirectional graphite/epoxy composite whose effective properties were determined in Example 1a. In addition, for each layer, the thickness (`THK`) and angle (`ANG`) in degrees must be specified. The thicknesses of the layers is important not only to provide the dimensions of the layers with respect to each other, but also to provide the overall thickness of the laminate. This overall thickness is important to the calculation of the laminate force and moment resultants during simulated applied loading. The angle specifies the orientation of each layer's local coordinate system relative to the laminate (global coordinate system). Note that the middle layer's thickness is twice that of the outer layers, resulting in a layup equivalent to four equal thickness layers.

Example problems dealing with other laminate configurations are presented within Section 3 of this Example Manual. For additional information on the code's laminate analysis capabilities, see the MAC/GMC 4.0 Keywords Manual Section 3 and the MAC/GMC 4.0 Theory Manual Section 3.

4) Loading: None

5) Damage and Failure: None

6) Output:

a) Output file print level (***PRINT**) [KM_6]:

```
NPL=-1
```

Print level: -1 (effective properties only) (`NPL=-1`)

b) x-y plots (***XYPLOT**): None

7) End of file keyword: (***END**)

Results

As in Example problem 1a, the results of this example are contained solely within the output file because only effective properties (as opposed to the response to applied loading) are determined. The lines below are taken directly from the output file. From the GMC repeating unit cell analysis of each layer, the effective stiffness matrix (`CG`), the effective compliance matrix (`CI`), the effective engineering moduli, and the effective CTEs are output (in local coordinates). Then, from the lamination theory analysis, the laminate axial stiffness (`A`), coupling stiffness (`B`), and bending stiffness (`D`) matrices are determined, as are the effective laminate engineering constants. Examining these results, it is clear that (in the local coordinates of each layer) the effective properties of each layer are identical to each other and the unidirectional composite results from Example 1a. The laminate results show that, for this symmetric cross-ply laminate, there is no coupling between extension and bending (`B` matrix is zero) and the laminate normal and shear behavior is decoupled (A_{16}, A_{26}, D_{16}, and D_{26} are close to zero). Finally, the effective (apparent) elastic moduli of the laminate in the x and y directions are identical as expected.

```
-------------------- EFFECTIVE PROPERTIES AT TEMPERATURE =    21.00 ---------------------------

                     * FOR LAYER NUMBER  1 *

 CG - Effective/Macro Stiffness Matrix

    0.2571E+12    0.4500E+10    0.4500E+10
    0.4500E+10    0.7828E+10    0.3707E+10
    0.4500E+10    0.3707E+10    0.7828E+10
                                             0.1916E+10
                                                           0.4166E+10
                                                                         0.4166E+10

 CI - Effective/Macro Compliance Matrix

    0.3944E-11   -0.1539E-11   -0.1539E-11
   -0.1539E-11    0.1653E-09   -0.7738E-10
   -0.1539E-11   -0.7738E-10    0.1653E-09
                                             0.5219E-09
                                                           0.2400E-09
                                                                         0.2400E-09

 Effective Engineering Moduli

     E11S= 0.2535E+12
     N12S= 0.3901
     E22S= 0.6050E+10
     N23S= 0.4682
     E33S= 0.6050E+10
     G23S= 0.1916E+10
     G13S= 0.4166E+10
     G12S= 0.4166E+10

 Effective Thermal Expansion Coefficients

   -0.4724E-06    0.2663E-04    0.2663E-04


-------------------- EFFECTIVE PROPERTIES AT TEMPERATURE =    21.00 ---------------------------

                     * FOR LAYER NUMBER  2 *

 CG - Effective/Macro Stiffness Matrix

    0.2571E+12    0.4500E+10    0.4500E+10
    0.4500E+10    0.7828E+10    0.3707E+10
    0.4500E+10    0.3707E+10    0.7828E+10
                                             0.1916E+10
                                                           0.4166E+10
                                                                         0.4166E+10

 CI - Effective/Macro Compliance Matrix

    0.3944E-11   -0.1539E-11   -0.1539E-11
   -0.1539E-11    0.1653E-09   -0.7738E-10
   -0.1539E-11   -0.7738E-10    0.1653E-09
                                             0.5219E-09
                                                           0.2400E-09
                                                                         0.2400E-09

 Effective Engineering Moduli

     E11S= 0.2535E+12
     N12S= 0.3901
```

```
     E22S= 0.6050E+10
     N23S= 0.4682
     E33S= 0.6050E+10
     G23S= 0.1916E+10
     G13S= 0.4166E+10
     G12S= 0.4166E+10

 Effective Thermal Expansion Coefficients

   -0.4724E-06    0.2663E-04    0.2663E-04


-------------------- EFFECTIVE PROPERTIES AT TEMPERATURE =    21.00 ---------------------------


                      * FOR LAYER NUMBER  3 *


 CG - Effective/Macro Stiffness Matrix

    0.2571E+12    0.4500E+10    0.4500E+10
    0.4500E+10    0.7828E+10    0.3707E+10
    0.4500E+10    0.3707E+10    0.7828E+10
                                              0.1916E+10
                                                            0.4166E+10
                                                                          0.4166E+10

 CI - Effective/Macro Compliance Matrix

    0.3944E-11   -0.1539E-11   -0.1539E-11
   -0.1539E-11    0.1653E-09   -0.7738E-10
   -0.1539E-11   -0.7738E-10    0.1653E-09
                                              0.5219E-09
                                                            0.2400E-09
                                                                          0.2400E-09

 Effective Engineering Moduli

     E11S= 0.2535E+12
     N12S= 0.3901
     E22S= 0.6050E+10
     N23S= 0.4682
     E33S= 0.6050E+10
     G23S= 0.1916E+10
     G13S= 0.4166E+10
     G12S= 0.4166E+10

 Effective Thermal Expansion Coefficients

   -0.4724E-06    0.2663E-04    0.2663E-04


---------------------- LAMINATE RESULTS AT TEMPERATURE =    21.00 ------------------------------
-


  Laminate Axial Stiffness Matrix [A]

      1.303E+11    2.369E+09   -4.748E-01
      2.369E+09    1.303E+11   -2.500E+01
     -4.748E-01   -2.500E+01    4.166E+09

  Laminate Coupling Stiffness Matrix [B]

      0.000E+00    0.000E+00    0.000E+00
      0.000E+00    0.000E+00    0.000E+00
      0.000E+00    0.000E+00    0.000E+00
```

```
Laminate Bending Stiffness Matrix [D]

    3.093E+09    1.974E+08   -6.924E-02
    1.974E+08    1.862E+10   -3.646E+00
   -6.924E-02   -3.646E+00    3.472E+08

Laminate Engineering Constants (only valid for symmetric laminates)

   Exx=  1.302E+11
   Nxy=  1.819E-02
   Eyy=  1.302E+11
   Gxy=  4.166E+09
```